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Distributed Information Fusion through Advanced Multi-Agent Control

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14. ABSTRACT
Distributed consensus in the Wasserstein metric space of probability measures was the primary topic of investigation under this project. Convergence of each agent's (or nodes) measure to a common probability measure is proven under a weak network connectivity condition. The common measure reached at each agent is one minimizing a weighted sum of its Wasserstein distance to all initial agent measures. This measure is known as the Wasserstein barycenter. Special cases involving Gaussian measures, empirical measures, and time-invariant network topologies are considered, where convergence rates and average-consensus results are given. This algorithm has potential applicability in computer vision, machine learning and distributed estimation, etc. A number of other topics in distributed and Monte-Carlo estimation were also considered including: distributed information fusion under unknown correlations; large-scale sequential Monte-Carlo methods; optimal controller approximation via Monte-Carlo methods; score and information matrix approximation via sequential Monte-Carlo methods.

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Distributed Information Fusion via Control in the Space of Probability Measures

Among other related directions, the *primary topic* of this project was to consider distributed consensus and information fusion in the space of probability measures. In particular:

> **Distributed consensus in the Wasserstein metric space of probability measures was introduced under this project. Convergence of each agent's (or node's) measure to a common probability measure is proven under a weak network connectivity condition. The common measure reached at each agent is one minimizing a weighted sum of its Wasserstein distance to all initial agent measures. This measure is known as the Wasserstein barycenter. Special cases involving Gaussian measures, empirical measures, and time-invariant network topologies are considered, where convergence rates and average-consensus results are given. This algorithm has potential applicability in computer vision, machine learning and distributed estimation, etc.**

This work and its applications are found in the following papers and technical articles:

- Adrian N. Bishop and Arnaud Doucet, "Consensus in the Wasserstein Metric Space of Probability Measures", 2015, arxiv.org/abs/1404.0145
- Adrian N. Bishop and Arnaud Doucet, "Distributed Nonlinear Consensus in the Space of Probability Measures", In Proceedings of the 19th International Federation of Automatic Control World Congress, August, 2014, Cape Town, South Africa.

These papers are attached in the appendices[1].

Focus of the above Wasserstein (barycenter) consensus algorithm to the problem of distributed combining (randomly) sampled probability measures (so called empirical measures) was explored in the following articles (attached in the appendices):

- Thibaut Lienart, Yee Whye The and Arnaud Doucet, "Expectation Particle Belief Propagation", In Proceedings of the Advances in Neural Information Processing Systems 28 (NIPS), December, 2015, Montreal, Canada.
- Adrian N. Bishop, "Information fusion via the Wasserstein barycenter in the space of probability measures: Direct fusion of empirical measures and Gaussian fusion with unknown correlation", In Proceedings of the 17th International Conference on Information Fusion (FUSION), July, 2014, Salamanca, Spain.
- Isaac L. Manuel and Adrian N. Bishop, "Distributed Monte Carlo Information Fusion and Distributed Particle Filtering", In Proceedings of the 19th International Federation of Automatic Control World Congress (IFAC WC), August, 2014, Cape Town, South Africa.
- Adrian N. Bishop, "Gossip-based distributed data fusion of empirical probability measures", In Proceedings of the 2014 IEEE Workshop on Statistical Signal Processing (SSP), June, 2014, Brisbane, Australia.

Research, development and collaborative work, on the fusion of empirical measures and on fusion methods that are robust to unknown correlations and double counting of information, is ongoing through collaboration and partnerships with researchers at the Air Force Research Lab (AFRL). This ongoing work is inspired by, and extends some of the work noted above.

[1]All papers referenced in this report acknowledge the support of the US Air Force Office of Scientific Research (AFOSR) through the Asian Office of Aeronautic Research and Development (AOARD).

A number of other topics were investigated during this project that may be considered related but also tangent to the primary topic (of Wasserstein barycenters and consensus in the space of probability measures). In particular, focus on empirical estimation (e.g. Monte Carlo methods) for control and estimation (including in the computation of Wasserstein barycenters and consensus) was a common thread throughout this project.

In the following two papers, the focus was on methods for computing the first and second derivatives of optimal filters (e.g. on approximating the corresponding information matrix):

- Pierre Del Moral, Arnaud Doucet and Sumeetpal S. Singh, “Uniform Stability of a Particle Approximation of the Optimal Filter Derivative”, SIAM Journal on Control and Optimization, 2015, Vol 53, No 3, pages: 1278–1304, dx.doi.org/10.1137/140993703
- Arnaud Doucet, Pierre E. Jacob and Sylvain Rubenthaler, “Derivative-Free Estimation of the Score Vector and Observed Information Matrix with Application to State-Space Models”, 2015, arxiv.org/abs/1304.5768

These papers are attached to the appendices. The importance of approximating the relevant derivatives (and in understanding the stability of these approximations rigorously) follows from their wide-spread use in parameter estimation and filtering; e.g. in determining the parameters of a multi-target model while simultaneously estimating the target states. One obvious justification for this wide-spread use is the prevalence of maximum likelihood methods and their common reliance on the use of derivatives.

The problem of approximating filtering probabilities in high-dimensional state-space models is investigated in the following two papers (attached in the appendices):

- Francesco Bertoli and Adrian N. Bishop, “Reducing the Bias in Blocked Particle Filtering for High-Dimensional Systems”, 2014, arxiv.org/abs/1407.0220
- Francesco Bertoli and Adrian N. Bishop, “Adaptively Blocked Particle Filtering with Spatial Smoothing in Large-Scale Dynamic Random Fields”, 2014, arxiv.org/abs/1406.0136

The problem of approximating the optimal controller in nonlinear stochastic optimal control problems is investigated in the following two papers (attached in the appendices):

- Francesco Bertoli and Adrian N. Bishop, “Nonlinear Stochastic Receding Horizon Control: Stability, Robustness and Monte Carlo Methods for Control Approximation”, 2015, arxiv.org/abs/1507.02780
- Francesco Bertoli and Adrian N. Bishop, “Monte Carlo Methods for Controller Approximation and Stabilization in Nonlinear Stochastic Optimal Control”, In Proceedings of the 17th IFAC Symposium on System Identification (SYSID), October, 2015, Beijing, China.

High-dimensional filtering and control in complex (e.g. nonlinear stochastic) models is challenging in practice since the error of “most” approximation methods is somehow exponentially dependent on model dimension. For example, naïve Monte Carlo integration displays an error that is typically exponential in the dimension of the integral. The filtering and control papers above look at addressing this dependency by “cutting” up the model into components of lower-dimension and accepting a bias as a result. The total error may be subsequently be reduced.

Appendix: Articles and Technical Reports

The following articles are now given in order:

1. Adrian N. Bishop and Arnaud Doucet, “Consensus in the Wasserstein Metric Space of Probability Measures”, 2015, arxiv.org/abs/1404.0145
2. Adrian N. Bishop and Arnaud Doucet, “Distributed Nonlinear Consensus in the Space of Probability Measures”, In Proceedings of the 19th International Federation of Automatic Control World Congress, August, 2014, Cape Town, South Africa.
3. Thibaut Lienart, Yee Whye The and Arnaud Doucet, “Expectation Particle Belief Propagation”, In Proceedings of the Advances in Neural Information Processing Systems 28 (NIPS), December, 2015, Montreal, Canada.
4. Adrian N. Bishop, “Information fusion via the Wasserstein barycenter in the space of probability measures: Direct fusion of empirical measures and Gaussian fusion with unknown correlation”, In Proceedings of the 17th International Conference on Information Fusion (FUSION), July, 2014, Salamanca, Spain.
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6. Adrian N. Bishop, “Gossip-based distributed data fusion of empirical probability measures”, In Proceedings of the 2014 IEEE Workshop on Statistical Signal Processing (SSP), June, 2014, Brisbane, Australia.
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8. Arnaud Doucet, Pierre E. Jacob and Sylvain Rubenthaler, “Derivative-Free Estimation of the Score Vector and Observed Information Matrix with Application to State-Space Models”, 2015, arxiv.org/abs/1304.5768
9. Francesco Bertoli and Adrian N. Bishop, “Reducing the Bias in Blocked Particle Filtering for High-Dimensional Systems”, 2014, arxiv.org/abs/1407.0220
10. Francesco Bertoli and Adrian N. Bishop, “Adaptively Blocked Particle Filtering with Spatial Smoothing in Large-Scale Dynamic Random Fields”, 2014, arxiv.org/abs/1406.0136
11. Francesco Bertoli and Adrian N. Bishop, “Nonlinear Stochastic Receding Horizon Control: Stability, Robustness and Monte Carlo Methods for Control Approximation”, 2015, arxiv.org/abs/1507.02780
12. Francesco Bertoli and Adrian N. Bishop, “Monte Carlo Methods for Controller Approximation and Stabilization in Nonlinear Stochastic Optimal Control”, In Proceedings of the 17th IFAC Symposium on System Identification (SYSID), October, 2015, Beijing, China.